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FREE ENTRY WITH COST REDUCING INVESTMENT: A NOTE

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ABSTRACT

This paper extends Mankiw & Whinston´s (86) characterization of excessive entry to the case where firms, in addition to producing, can also make a deterministic cost reduction investment, either before, or at the same time as production.

Key Words: Excessive Entry, Cost Reducing Investment

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1 Introduction

This paper extends Mankiw & Whinston's (86) characterization of excessive entry to the case where firms, in addition to producing, can also invest in marginal production cost reduction[1]. The deterministic investment can occur before, or at the same time as production. The problem of excessive in the presence of investment arises for example in the Research and Development literature.

Mankiw & Whinston (86) (M-W) showed for the *no investment* case, that in homogeneous product markets with fixed costs, there is a bias for excessive entry[2]. They analyzed a two stage game, where first firms decide whether to enter the industry, and next play an oligopolistic game, and compared the number entry of firms that enter the market in a free-entry equilibrium, with the second-best[3] welfare-maximizing number of firms.

The appeal of M-W's characterization of excessive entry is that it consists of two intuitive and easy to check conditions. The first is ***imperfect competition***: the equilibrium price exceeds marginal cost. The second is the ***business stealing effect***: the equilibrium strategic response of existing firms to new entry, is to lower output.

With fixed costs, even if there is free entry, an industry will not consist of a large number of infinitesimal firms, but rather of an oligopoly. If a firm[4] by entering the market causes other imperfectly competitive firms to reduce their output, and if the other firms have positive profit margins, they lose income. The entrant's contribution to social surplus equals his profit, less the social value of the output lost due to the output reduction he imposes to other firms. Thus, entry is more valuable to the entrant than to society. The welfare loss of free entry disappears as the fixed cost approaches zero.

Although investment expenditures are choice variables for firms, once made they are fixed costs for production. Firms must earn quasi rents in equilibrium to cover them, leading to an oligopolistic market. Also, since by rising its investment a firm can improve its competitiveness in the product market, imperfectly competitive firms may not minimize total (production plus investment) costs, but rather use investment strategically. This strategic effect can generate an additional inefficiency: socially excessive investment.

I show that M&W's characterization also applies when firms invest. I make three observations. First, when investment and production are simultaneous, ***imperfect competition*** and the ***business stealing effect*** are

[1] For example: process Research and Development, or investment in capital that rises the marginal productivity of labor.
[2] With product heterogeneity, the bias can be reversed, since entry entails the creation of more product diversity, which is socially valuable.
[3] That is, given the firms' oligopolistic behavior.
[4] That is small and thus has no first order effects on price, which is somewhat inconsistent with the existence of fixed costs.

still sufficient for excessive entry. When total costs are minimized, the characterization of excessive entry reduces to the no investment case, whether investment occurs before or precedes production. And when output and investment are simultaneous, firms minimize the cost of producing their chosen output. Second, for a Cournot oligopoly, without investment, or with simultaneous investment and production, the ***business stealing effect*** is equivalent to ***strategic substitutability***[5] of the firms' outputs. Third, for a Cournot oligopoly where investment precedes production, ***strategic substitutability*** of the firms' outputs, in addition to the ***business stealing effect*** are sufficient for excessive entry. For this model the ***business stealing effect*** and ***strategic substitutability*** are not equivalent.

Previous analysis of excessive entry when firms also invest was limited to specific models. Tandon (84) showed, for Dasgupta & Stiglitz's (80) model of a Cournot oligopoly with *simultaneous* investment and production, that when demand is linear[6], and marginal costs constant, welfare falls with the number of firms, at the free entry level. Since a fall in the number of firms from the free entry level rises welfare, excessive entry follows. He questioned if the result depended on the functional forms used[7]. But, since investment and production are simultaneous, excessive entry depends only on the ***business stealing effect***. Okuno-Fujiwara & Suzumura (93) showed for a Cournot oligopoly where investment precedes production, that when the elasticity of the slope of the inverse demand function is bounded below, the firms' outputs are ***strategic substitutes***, and marginal production costs constant, welfare falls with the number of firms at the free entry level.

In section 2 I introduce the models. In section 3 I analyze the case where investment and production occur simultaneously, and in section 4 the case where investment precedes production.

2 The Model

In this section I describe two models where firms decide whether to enter the industry, and choose investment and production levels. In the first, the ***simultaneous model***, investment and production are chosen simultaneously. In the second, the ***sequential model***, production is chosen after investment.

2.1 Common Notation and Assumptions

In this sub-section I introduce notation and assumptions common to both models.

[5] Firms' outputs are ***strategic substitutes***, when a rise in a rival's output reduces the marginal profitability of a firm's output (Bulow, Geanakoplos & Klemperer (85)). This occurs, for example, in a Cournot oligopoly with downward sloping best response functions.
[6] Dasgupta & Stiglitz's (80) considered an isoelastic inverse demand function.
[7] Tandon (84), footnote 14.

Consider a market for a homogeneous good that opens for one period.

There is a unit mass of identical consumers. Consumers have an utility function $U(q,m) = u(q) + m$, where q is the quantity consumed of the good and m is money, of the form $U(.,.): \Re_0^+ \times \Re_0^+ \longrightarrow \Re$; $u(.)$ is twice differentiable, strictly increasing and concave. The inverse demand function, $P(Q)$, where Q is the industry's output, is of the form $P(.): \Re^+ \longrightarrow \Re^+$, and is once differentiable and decreasing.

There is a (countable) infinity of identical firms. Firm i's production cost function $C(q_i,e_i)$, where q_i and e_i are firm i's production and investment levels, is of the form $C(.,.): \Re_0^+ \times \Re_0^+ \longrightarrow \Re_0^+$, is once differentiable and increasing in output, twice differentiable, decreasing and convex in investment, and the marginal production cost is decreasing in investment: $C_q > 0, C_e < 0, C_{ee} > 0, C_{qe} < 0$.

2.2 The Simultaneous Model

In this sub-section I introduce the simultaneous model. The simultaneous model consists of two stages (figure 1). In stage 1, firms decide simultaneously whether to enter the industry; if a firm decides to enter it must pay a fixed cost $F > 0$. In stage 2, firms simultaneously determine investment and production levels[8].

[Insert figure 1 here]

Stage 2 is some oligopolistic game not modeled explicitly. However, I assume that given any number of firms, it has a unique symmetric equilibrium in pure strategies. Denote a firm's stage 2 equilibrium investment and output, given that n firms entered the industry, by $e(n)$ and $q(n)$. A firm's stage 2 equilibrium ***payoff*** is: $\pi(n) := P(nq(n))q(n) - C(q(n),e(n)) - e(n)$. Stage 2 ***equilibrium*** is $\{e(n),q(n)\}$.

A firm's stage 1 ***strategy*** is an *in* or *out* decision. A firm's ***payoff*** for the whole game is $\Pi(n;F) := \pi(n) - F$. A ***free entry equilibrium*** is defined as an asymmetric[9] subgame perfect equilibrium in pure strategies of the whole game, which satisfies the free entry condition:

$$\Pi(n^f+1) < 0 \leq \Pi(n^f).$$

where n^f is the free entry equilibrium number of firms. A ***free entry equilibrium*** is $\{n^f,e(n^f),q(n^f)\}$.

For both the present and the following model I assume that the free entry equilibrium number of firms exactly satisfies the free entry condition (i.e., I assume away the integer constraint):

$$\Pi(n^f) = 0. \tag{1}$$

[8] I say determine rather than choose because firms might choose prices rather than quantities.
[9] The only symmetric equilibrium is in mixed strategies (Dixit & Shapiro (1986)).

2.3 The Sequential Model

In this sub-section I introduce the sequential model. The sequential model consists of three stages (figure 2). In stage 1, firms decide simultaneously whether to enter the industry; if a firm decides to enter it must pay fixed cost $F>0$. In stage 2, firms simultaneously choose an investment level. At the end of this stage the industry investment profile becomes common knowledge. In stage 3, firms simultaneously choose a production level.

[Insert figure 2 here]

Stages 2 and 3 are oligopolistic games, which are not modeled explicitly. However, I assume that: given any number of firms and investment profile, stage 3 has a unique symmetric equilibrium in pure strategies, and given any number of firms, stage 2 has a unique symmetric subgame perfect equilibrium in pure strategies, where the relevant game is defined by stages 2 and 3. Denote a firm's stage 3 equilibrium output, given that n firms entered the industry and the industry investment profile is $\mathbf{e} := (e_1,\dots,e_n)$ by $\bar{q}(e,n)$. A firm's stage 3 ***payoff*** is: $\pi(e,n) := P(n\bar{q}(e,n))\bar{q}(e,n) - C(\bar{q}(e,n),e) - e$. Denote a firm's stage 2 equilibrium investment level by $\bar{e}(n)$. A firm's stage 2 equilibrium ***payoff*** is: $\pi(n) := P(n\bar{q}(\bar{e}(n),n))\bar{q}(\bar{e}(n),n) - C(\bar{q}(\bar{e}(n),n),\bar{e}(n)) - \bar{e}(n)$. Stage 2's subgame perfect ***equilibrium*** is $\{\bar{e}(n), \bar{q}(\bar{e}(n),n)\}$.

A firm's stage 1 ***strategy*** is an *in* or *out* decision. A firm's ***payoff*** for the whole game is $\Pi(n;F) := \pi(n) - F$. A ***free entry equilibrium*** is defined as an asymmetric subgame perfect equilibrium in pure strategies of the whole game, which satisfies (1). A ***free entry equilibrium*** is $\{\bar{n}^f, \bar{e}(\bar{n}^f), \bar{q}(\bar{e}(\bar{n}^f),\bar{n}^f)\}$.

Denote a firm's equilibrium output in the sequential model by $\bar{q}(n) := \bar{q}(\bar{e}(n),n)$. Define $\partial\bar{q}/\partial e := (\partial\bar{q}_i/\partial e_i) + (n-1)(\partial\bar{q}_j/\partial e_i)$, then $d\bar{q}/dn = (\partial\bar{q}/\partial n) + (\partial\bar{q}/\partial e)(d\bar{e}/dn)$. Define $\Lambda := \partial^2\Pi/\partial q_j\partial q_i = P' + q_i P''$.

2.4 Additional Assumptions

In this sub-section I recall M-W's conditions for excessive entry and introduce an additional assumption.

M-W's conditions for excessive entry are:

(A.1) **(i)** $nq(n) \geq n'q(n'), \forall n>n'$

(ii) $\lim_{n\to\infty} nq(n) < \infty.$

(A.2) $q(n) \leq q(\bar{n}), \forall n>\bar{n}$

(A.3) $P(Q(n)) - \dfrac{\partial C(q(n),e(n))}{\partial q} \geq 0, \forall n$

The first, guarantees that the free entry equilibrium number of firms is well defined. The second, is the ***business stealing effect***. The third, is the ***imperfect competition*** requirement.

Next I introduce the additional assumption that firms' outputs are ***strategic substitutes***.

(A.4) $\Lambda < 0$

3 Analysis of the Simultaneous Model

In this section I present 2 results: first, for the simultaneous model ***strategic substitutability*** and ***imperfect competition*** are sufficient for excessive entry, second, for a simultaneous Cournot oligopoly, with or without investment, the ***business stealing effect*** is equivalent to ***strategic substitutability*** of the firms' outputs.

The socially optimal number of firms, n^*, solves:

$$\max_{n^*} W(n) = \int_0^{nq(n)} P(s)\,ds - nC(q(n),e(n)) - ne(n) - nF \tag{2}$$

whose necessary condition is:

$$\frac{dW(n^*)}{dn} = \Pi(n^*) + n^*\left(P - \frac{\partial C}{\partial q}\right)\frac{dq}{dn} - n^*\left(\frac{\partial C}{\partial e} + 1\right)\frac{de}{dn} = 0.$$

Comparison with the equivalent expression of the problem without investment, shows that investment generates the additional term $[(\partial C/\partial e) + 1](de/dn)n^*$, whose sign depends on $[(\partial C/\partial e) + 1]$, which has to do with whether total costs are minimized[10], and (de/dn), which has to do with how a firm's equilibrium investment varies with the number of firms in the industry. When total costs are minimized, the additional term disappears, since $\partial C/\partial e + 1 = 0$. Next I argue, that for the simultaneous model total costs are minimized.

In stage 2 firms either play non-cooperatively, or cooperatively, i.e., form a cartel. The cartel solves the monopoly problem, and then distributes costs and revenues among its members. And a monopolist minimizes the cost of producing its chosen output[11]. Furthermore, the timing of investment is irrelevant. For the non-cooperative oligopoly, since output and investment are determined simultaneously, whatever its opponents' investment and production strategies may be, a firm best responds by minimizing the cost of producing its chosen output. Hence, in every Nash equilibria, costs are minimized, given the production level. Summarizing:

[10] The necessary condition for cost minimization problem, $\min_e C(q,e) + e$ s.t. $q = \bar{q}$, is: $C_e + 1 = 0$.
[11] Investment is not used strategically because the cartel's or the monopolist's problem is a *decision theoretic* problem.

Lemma 1: For the simultaneous model, total costs are minimized. ¶

When total costs are minimized, an argument identical to M-W's for the no investment case, shows that conditions (A1)-(A3) are sufficient for excessive entry[12], leading to the following proposition.

Proposition 1: For the simultaneous model (A1)-(A3) are sufficient for socially excessive entry. ¶

Next I present some examples that illustrate the previous discussion.

Example 1: **The Bertrand-Edgeworth Model: Absence of Business Stealing.** Suppose firms have constant marginal costs, and face a capacity constraint $q_i \leq \bar{q} = 1$. Assume $\bar{q}$ is invariant to investment and $P(1) > C_q(1,e_i)$, $\forall e_i$. In equilibrium: each firm produces at capacity, the industry output equals the number of firms in the industry, the price clears the market, and firms enter the industry if $P(n) - C(1,e_i) - e_i \geq 0$. Additional entry entails no business loss for the firms in the industry, and investment' s necessary condition is: $- C_e(q_i,e_i) - 1 = 0$[13].¶

Example 2 : **The Cartel.** A cartel's problem is: $\max_{Q\,E} \; P(Q)Q - C(Q,E) - E$, where $E := ne$ is the cartel investment level. Investment's necessary condition is: $-C_e - 1 = 0$. ¶

Example 3: **The Simultaneous Cournot Oligopoly.** A Cournot oligopolist's problem is: $\max_{q\;e} \; P(Q)q_i - C(q_i,e_i) - e_i$. The necessary condition for investment is: $-C_e - 1 = 0$. ¶

Next I argue that for a Cournot oligopoly, with or without investment, the ***business stealing effect*** is equivalent to ***strategic substitutability*** of the firms' outputs. Applying the implicit function theorem to the necessary conditions of both mentioned cases gives (see appendix A for details):

$$\text{sign}\{dq(n)/dn\} = \text{sign}\{\Lambda\} = \text{sign}\{P' + qP''\}$$

Thus, the ***business stealing effect*** is present, if the firms' outputs are ***strategic substitutes,*** $\Lambda < 0$, which occurs if demand is not *too convex*: $(-P''/P') < 1/q$. Summarizing:

Proposition 2: For a simultaneous Cournot oligopoly, with or without investment, the ***business stealing effect*** is equivalent to ***strategic substitutability*** of the firms' outputs, i.e. **(A.2)** is equivalent to **(A.4)**. ¶

[12] (A1)-(A3) imply profit is non-increasing in n, and that: $0 = \Pi(n^f) = dW(n^*)/dn \leq \Pi(n^*)$. Given that profit is non-increasing in n: $n^* \geq n^f$.

[13] Notice that the timing of investment is irrelevant.

Propositions 1 and 2 address Tandon (84)'s concern of whether his result depended on the functional forms used. For his and Dasgupta & Stiglitz's (80) model, excessive entry depends on demand not being *too convex*.

4 Analysis of the Sequential Model

In this section I show that for the sequential Cournot oligopoly an additional condition is required for socially excessive entry: ***strategic substitutability*** of the firms' outputs.

The socially optimal number of firms, n^*, solves (2), whose necessary condition is now:

$$\frac{dW(n^*)}{dn} = \Pi(n^*) + n^*\left(P - \frac{dC}{dq}\right)\left[\frac{\partial\bar{q}}{\partial n} + \frac{\partial\bar{q}}{\partial e}\frac{d\bar{e}}{dn}\right] - n^*\left(\frac{\partial C}{\partial e} + 1\right)\frac{d\bar{e}}{dn} = 0$$

Again, when total costs are minimized the term associated with investment disappears, and conditions **(A.1)-(A.3)** are sufficient for excessive entry. Although that happens for a cartel, now it is not always in the firm's interest to minimize the cost of producing its chosen output. In the production stage, even if a firm wanted to readjust its investment, given its choice of output and the opponents' strategies, it could no longer do so, because the investment decision has already been made in the previous stage. This ***commitment effect*** that investment can have in the sequential model, creates an incentive for total costs to deviate from its minimum, towards socially excessive investments, as firms attempt to rise their competitiveness in the production stage (Brander & Spencer (83), Bulow, Geanakoplos, & Klemperer, (85)).

When total costs are not minimized, and there is over investment, the welfare impact of entry is potentially ambiguous. Entry has a negative impact due to the ***business stealing effect.*** But, if the entrant causes firms to reduce their investments, entry also has the positive impact of mitigating over investment. In that case, entry causes an aggregate investment contraction of $n(d\bar{e}/dn)$ and a rise in welfare of $-(C_e + 1)\, n(d\bar{e}/dn)$.

Next I show that, if in addition to the ***business stealing effect*** one assumes that the firms' outputs are ***strategic substitutes,*** excessive entry follows again. I will proceed in four steps (see appendix B for details).

First, applying the implicit function theorem to stage 3's necessary condition gives:

$$\text{sign}\{\partial\bar{q}\,(\bar{e}(n),n)/\partial n\} = \text{sign}\{\Lambda\}.$$

Thus $\partial\bar{q}/\partial n < 0$, under ***strategic substitutability.***

Second, applying the implicit function theorem to stage 2's necessary condition gives:

$$\text{sign}\{\partial\bar{q}_j(\bar{e}(n),n)/\partial e_i\} = \text{sign}\{\Lambda\}$$

Thus, $\partial\bar{q}_j/\partial e_i < 0$, under ***strategic substitutability.*** Also $\partial\bar{q}/\partial e := (\partial\bar{q}_i/\partial e_i) + (n-1)(\partial\bar{q}_j/\partial e_i) > 0$.

Third, inspection of the stage 2's necessary condition shows that if $\partial\tilde{q}_j/\partial e_i < 0$, then $(C_e + 1) > 0$.

Forth, the ***business stealing effect***, and $\partial\tilde{q}/\partial e > 0$, imply: $d\bar{e}/dn < -(\partial\tilde{q}/\partial e)/(\partial\tilde{q}/\partial e)$. Replacing on the necessary condition for welfare maximization it follows that:

$$\frac{dW(n^*)}{dn} < \Pi(n^*) + n^*\left(\frac{\partial C}{\partial e} + 1\right)\frac{\partial\tilde{q}/\partial n}{\partial\tilde{q}/\partial e} \tag{3}$$

and that the last term in (3) is negative. Excessive entry follows by M-W's argument. Summarizing:

Proposition 3: For a sequential Cournot oligopoly (A.1)-(A.4) are sufficient for socially excessive entry. ¶

Okuno-Fujiwara & Suzumura (93) assumed a class of inverse demand functions whose slope has an elasticity bounded below. This class of does not necessarily imply ***strategic substitutability***. However, the authors also assumed ***strategic substitutability***.

Condition (3) was derived for a Cournot oligopoly, but depends only on: the ***business stealing effect***, $\partial\tilde{q}/\partial n \geq 0$, $\partial\tilde{q}/\partial e > 0$, and $C_e + 1 \geq 0$, independently of the stages 2 and 3 oligopolistic games.

To clarify the importance of assuming both the ***business stealing effect***, and that the firms' outputs are ***strategic substitutes***, notice that given (see appendix B):

$$\operatorname{sign}\left(\frac{d\bar{e}}{dn}\right) = \operatorname{sign}\left\{-C_{eq}\frac{\partial\tilde{q}_i}{\partial n} + \tilde{q}_i P'\frac{\partial\tilde{q}_j}{\partial e_i} + (n-1)P'\frac{\partial\tilde{q}_i}{\partial n}\frac{\partial\tilde{q}_j}{\partial e_i} + (n-1)P'\tilde{q}_i\frac{\partial^2\tilde{q}_j}{\partial n\partial e_i} + (n-1)\tilde{q}_i\frac{\partial\tilde{q}_j}{\partial e_i}P''\frac{dQ}{dn}\right\}$$

and that $d\tilde{q}/dn := (\partial\tilde{q}/\partial n) + (\partial\tilde{q}/\partial e)\,(d\bar{e}/dn)$, it follows that for the Cournot sequential model, ***strategic substitutability***, is neither necessary nor sufficient for the ***business stealing effect***[14].

Appendix A

Consider the necessary condition for the firm's problem without investment:

$$\Pi_q = P(nq) + q\,P'(nq) - C_q(q,e) = 0$$

Applying the implicit function theorem: $dq(n)/dn = \left[-qA/(n+1)P' + nqP'' - C_{qq}\right]$. Since sufficiency requires $(n + 1)P' + nqP'' - C_{qq} < 0$, it follows that: $\operatorname{sign}\{dq(n)/dn\} = \operatorname{sign}\{A\}$.

Consider the system of the necessary conditions for the firm's problem with investment:

$$\Pi_q = P(nq) + q\,P'(nq) - C_q(q,e) = 0$$

$$\Pi_e = -C_e(q,e) - 1 = 0$$

Differentiating gives:

$$\begin{bmatrix} \Pi_{qq} & \Pi_{qe} \\ \Pi_{qe} & \Pi_{ee} \end{bmatrix}\begin{bmatrix} dq/dn \\ de/dn \end{bmatrix} = \begin{bmatrix} -\Pi_{qn} \\ -\Pi_{en} \end{bmatrix}$$

Applying Cramer's rule: $de/dn = \Pi_{en}/\Delta = qA/\Delta$, $dq/dn = \left(-\Pi_{qn}\Pi_{en}\right)/\Delta = qAC_{ee}/\Delta$, where $\Delta = \Pi_{qq}\Pi_{ee} - (\Pi_{qe})^2 = \Pi_{qq}C_{ee} - (C_{qe})^2 > 0$. Since C_{ee} is positive at the optimum[15]: $\operatorname{sign}\{dq(n)/dn\} = \operatorname{sign}\{de(n)/dn\} = \operatorname{sign}\{A\}$.

Appendix B

First, applying the implicit function theorem to stage 3's necessary condition:

$$\Pi_q = P(Q) + \tilde{q}\,P'(Q) - C_q(\tilde{q}, \tilde{e}) = 0$$

gives: $\partial\tilde{q}/\partial n = \left[-\tilde{q}A/(n+1)P' + n\tilde{q}P'' - C_{qq}\right]$. Sufficiency requires $(n + 1)P' + n\tilde{q}\,P'' - C_{qq} < 0$, thus: $\operatorname{sgn}\{\partial\tilde{q}/\partial n\} = \operatorname{sgn}\{A\}$.

Second, using the envelop theorem, stage 2's necessary condition is:

$$\frac{d\Pi_i}{de_i} = \frac{\partial\Pi_i}{\partial e_i} + (n-1)\frac{\partial\Pi_i}{\partial q_j}\frac{\partial q_j}{\partial e_i} = 0$$

Differentiating with respect to e_i and e_j gives:

$$\Pi_{ii}\frac{\partial\bar{q}_i}{e_i} + (n-1)\,\Pi_{ij}\frac{\partial\bar{q}_j}{\partial e_i} - C_{qe} = 0$$

$$\Pi_{ij}\frac{\partial\bar{q}_i}{e_i} + [nP' + (n-1)\tilde{q}\,P'' - C_{qq}]\frac{\partial\tilde{q}_j}{\partial e_i} = 0$$

Solving for $\partial\tilde{q}_i/\partial e_i$ and $\partial\tilde{q}_j/\partial e_i$ gives: $(\partial\tilde{q}_i/\partial e_i) = \left\{C_{qe}\left[nP' + (n-1)\tilde{q}P'' - C_{qq}\right]/H\right\}$ and $(\partial\tilde{q}_j/\partial e_i) = \left[C_{qe}A/H\right]$, where $H = (P'-C_{qq})\,[(n+1)P' + n\tilde{q}\,P'' - C_{qq}] > 0$. Thus: $(\partial\tilde{q}_i/\partial e_i) + (n-1)(\partial\tilde{q}_j/\partial e_i) = 1/\left[(n+1)P' + n\tilde{q}P'' - C_{qq}\right] > 0$, if sufficiency holds, and $sign\{\tilde{q}_j(\bar{e}(n), n)/\partial e_i\} = sign\{A\}$.

Third, stage 2's necessary condition, given $\partial\Pi_i/\partial q_j < 0$, and $\partial\tilde{q}_j/\partial e_i < 0$, then $\partial\Pi_i/\partial e_i = -(C_e + 1) < 0$.

Fourth, if $d\bar{e}/dn < -(\partial\tilde{q}/\partial n)/(\partial\tilde{q}/\partial e)$, then:

$$\frac{dW(n)}{dn} = \Pi(n) + n\left(P - \frac{dC}{dq}\right)\left[\frac{\partial\tilde{q}_i}{\partial n} + \frac{\partial\tilde{q}_i}{\partial e}\frac{d\bar{e}}{dn}\right] - n\left(\frac{\partial C}{\partial e} + 1\right)\frac{d\bar{e}}{dn} = \Pi(n) + n\left(P - \frac{dC}{dq}\right)\frac{\partial\tilde{q}_i}{\partial n} + n\left(P - \frac{dC}{dq}\right)\frac{\partial\tilde{q}_i}{\partial e}\frac{d\bar{e}}{dn} - n\left(\frac{\partial C}{\partial e} + 1\right)\frac{d\bar{e}}{dn} <$$

$$\Pi(n) + n\left(P - \frac{dC}{dq}\right)\frac{\partial\tilde{q}_i}{\partial n} - n\left(P - \frac{dC}{dq}\right)\frac{\partial\tilde{q}_i}{\partial e}\frac{\partial\tilde{q}_i/\partial n}{\partial\tilde{q}_i/\partial e} + n\left(\frac{\partial C}{\partial e} + 1\right)\frac{\partial\tilde{q}_i/\partial n}{\partial\tilde{q}_i/\partial e} = \Pi(n) + n\left(\frac{\partial C}{\partial e} + 1\right)\frac{\partial\tilde{q}_i/\partial n}{\partial\tilde{q}_i/\partial e}$$

thus

$$\frac{dW(n^*)}{dn} < \Pi(n^*) + n^*\left(\frac{\partial C}{\partial e} + 1\right)\frac{\partial\tilde{q}_i/\partial n}{\partial\tilde{q}_i/\partial e}$$

[14] As Okuno-Fujiwara & Suzumura (93) observed, in general, strategic substitutability of the firm's outputs does not even imply strategic substitutability of the firms' investments.

[15] The positivity of C_{ee}, aside from being a concavity requirement needed fro the firm's problem to be well behaved, can also be interpreted as decreasing returns to cost reducing.

Applying the implicit function theorem to stage 2's necessary condition gives: $\left(d\tilde{e}_i(n)/dn\right) = -\Pi_{e_i n}/\Pi_{e_i e_i}$. Sufficiency requires that $\Pi_{e_i e_i} < 0$ thus: $sign(d\tilde{e}_i(n), dn) = sign(\Pi_{e_i n})$, where: $\Pi_{e_i n} = -C_{eq}\left(\partial\tilde{q}_i/\partial n\right) + \tilde{q}_i P'\left(\partial\tilde{q}_j/\partial e_i\right) + (n-1)P'\left(\partial\tilde{q}_i/\partial n\right)\left(\partial\tilde{q}_j/\partial n\right) + (n-1)\tilde{q}_i P'\left(\partial^2\tilde{q}_j/\partial n\partial e_i\right) + (n-1)\tilde{q}_i\left(\partial\tilde{q}_j/\partial e_i\right)P''\left(\partial Q/\partial n\right)$.

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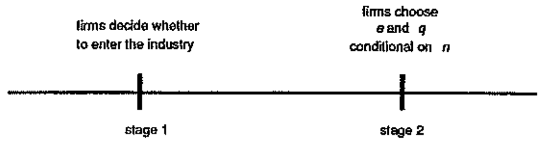
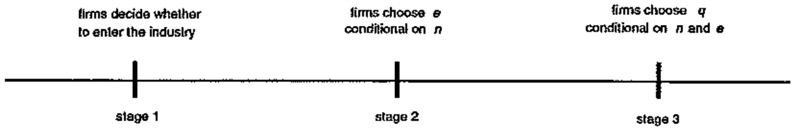


Simultaneous Model



Sequential Model